Question	Scheme	Marks	AOs
1(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0\Rightarrow y=-\frac{1}{2}x+...\Rightarrow -\frac{1}{2}\to 2$	B1	2.2a
	$m_N=2\Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0\Rightarrow x=...,\ y=...$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(\text{"5"}-\text{"}\frac{6}{5}\text{"}\right)^2+\left(\text{"4"}+\text{"}\frac{18}{5}\text{"}\right)^2}-\text{"3"}$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
			(8 marks)

Notes

(a)

M1: Attempts to complete the square for both x and y terms $(x\pm5)^2 .... (y\pm4)^2$ which may be implied by a centre of (±5, ±4)

A1: Centre (5, 4)

A1: Radius 3

(b)

B1: Deduces the gradient of the perpendicular to l is 2. May be seen in the equation for the perpendicular line to l

M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of l
Look for $y-\text{"4"}=\text{"2"}(x-\text{"5"})$ where $(5,4)$ is their centre being solved simultaneously with the equation of l
Do not be concerned with the mechanics of their rearrangement when solving simultaneously.
Many are finding the y-intercept of l $(0,-3)$ which is M0

A1: $\left(\frac{6}{5}, -\frac{18}{5}\right)$ or equivalent eg $(1.2, -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between *l* and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5}, \ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left(\text{"}\frac{25-3\sqrt{5}}{5}\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2 + \left(\text{"}\frac{20-6\sqrt{5}}{5}\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|\text{"}5\text{"}\times\text{"}1\text{"}+\text{"}4\text{"}\times\text{"}2\text{"}+\text{"}6\text{"}|}{\sqrt{\text{"}1\text{"}^2+\text{"}2\text{"}^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}} \left(=\frac{19}{\sqrt{5}}\right)$

dM1: Distance $= \text{"}\frac{19\sqrt{5}}{5}\text{"} - 3$

A1: $\frac{19\sqrt{5}-15}{5}$

Question	Scheme	Marks	AOs
2 (a)	$x^2+y^2-6x+10y+k=0$		
	$(x-3)^2+(y+5)^2 \pm ... = ...$	M1	1.1b
	Centre $(3,-5)$	A1	1.1b
		(2)	
(b)	Deduces that $k=9$ is a critical point	B1ft	2.2a
	Recognises that radius > 0 $"9"+"25"-k>0$	M1	3.1a
	$9<k<34$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: For sight of $(x\pm 3)^2 \pm (y\pm 5)^2 \pm ... = ...$ or one coordinate for centre from $(\pm 3, \pm 5)$

A1: Centre $(3,-5)$

(b)

B1ft: Deduces that $k...9$ is a critical point. Allow this to come from their $("5")^2$ Condone $\frac{36}{4}$

Note that this might come from setting $y=0$ and using the discriminant on $x^2-6x+k=0$

M1: $(x\pm 3)^2+(y\pm 5)^2=("3")^2+("5")^2-k$ **and** recognises that the radius^2 must be positive so $("3")^2+("5")^2-k>0$ but condone $("3")^2+("5")^2-k\geq 0$

$k<34$ **or** $k\leq 34$ would imply this method mark.

Note: they may have incorrectly calculated $("3")^2+("5")^2$ in (a) so allow their value for this in place of $("3")^2+("5")^2$ as long as the intention is clear.

A1: $9<k<34$ but condone $9<k\leq 34$. Allow inequalities to be separate, i.e., $k>9, k<34$

Set notation may be seen $\{k:k>9\}\cap\{k:k<34\}$ **or** $k\in(9,34)$

Condone $\{k:k>9\}\cap\{k:k\leq 34\}$ **or** $k\in(9,34]$ **or** $k>9$ and $k\leq 34$

Must not be combined incorrectly, e.g., $\{k:k>9\}\cup\{k:k<34\}$

Question	Scheme	Marks	AOs
3(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k\Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0\Rightarrow(6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2 . For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0\Rightarrow(x-a)^2+(y-b)^2=k\Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,-2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k. See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2+y^2-10x+4y+11=0\Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3\Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x\Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.

M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y\frac{dy}{dx} - 10 + 4\frac{dy}{dx} = 0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for l $y = 3x + k, x + 3y = 1$ $\Rightarrow x = ...$ and $y = ...$ in terms of k	M1	3.1a
	$x = \frac{-3k-1}{10},\ y = \frac{k-3}{10},\ x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow k = ...$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y = 3x + k \Rightarrow m = 3 \Rightarrow m_r = -\frac{1}{3}$	M1
	$y + 2 = -\frac{1}{3}(x-5)$	A1
	$(x-5)^2 + (y+2)^2 = 18,\ y + 2 = -\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2 = 18 \Rightarrow x = ...$ or $\Rightarrow 10(y+2)^2 = 18 \Rightarrow y = ...$	M1
	$x = \frac{25 \pm 9\sqrt{5}}{5},\ y = \frac{-10 \pm 3\sqrt{5}}{5},\ k = y - 3x \Rightarrow k = ...$	M1
	$k = -17 \pm 6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

M1: Solves simultaneously to find x or y

Alternatively solves "$y = -\frac{1}{3}x - \frac{1}{3}$" and $y = 3x + k$ to get x in terms of k which they substitute in $x^2 + (3x+k)^2 - 10x + 4(3x+k) + 11 = 0$ to form an equation in k.

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values

Question	Scheme	Marks	AOs
4 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=...$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
		(5 marks)	

Notes:

(a)(i)

M1: Attempts to complete the square on **both** *x* and *y* terms.

Accept $(x\pm5)^2+(y\pm8)^2=...$ or imply this mark for a centre of $(\pm5,\pm8)$

Condone $(x\pm5)^2....(y\pm8)^2=...$ where the first ... could be , or even –

A1: Correct centre $(5,-8)$.

Accept without brackets. May be written $x=5, y=-8$

(a)(ii)

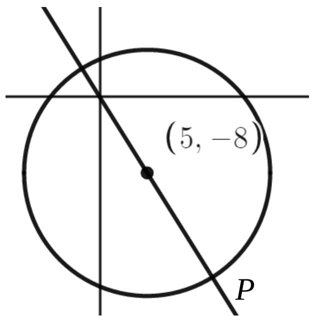
A1: 13. The M mark must have been awarded, so it can be scored following a centre of $(\pm5,\pm8)$.

Do not allow for $\sqrt{169}$ or ±13

(b)

M1: Attempts $\sqrt{"5"^2+"8"^2}+"13"$ for their centre $(5,-8)$ and their radius 13.

Award when this is given as a decimal, e.g. 22.4 for correct centre and radius. Look for $\sqrt{a^2+b^2}+r$ where centre is $(\pm a,\pm b)$ and radius is r



A1ft: $13+\sqrt{89}$ Follow through on their $(5,-8)$ and their 13 leading to an exact answer. ISW for example if they write $13+\sqrt{89}=22.4$

....

There are more complicated attempts which could involve finding *P* by solving $y="-\frac{8}{5}x"$ and $x^2+y^2-10x+16y=80$ simultaneously and choosing the coordinate with the greatest modulus. The method is only scored when the distance of the largest coordinate from *O* is attempted. Such methods are unlikely to result in an exact value but can score 1 mark for the method. Condone slips

FYI. Solving $y=-\frac{8}{5}x$ and $x^2+y^2-10x+16y=80 \Rightarrow 89x^2-890x-2000=0 \Rightarrow P=(11.89,-19.02)$

Hence $OP=\sqrt{"11.89"^2+"19.02"^2}\ (=22.43)$ scores M1 A0 but $OP=\sqrt{258+26\sqrt{89}}$ is M1 A1

Question	Scheme	Marks	AOs
5(a)(i)	Centre $(-3k,\ k)$	B1	2.2a
(ii)	$(x+3k)^2-9k^2+(y-k)^2-k^2+7=0\Rightarrow(x+3k)^2+(y-k)^2=...$	M1	1.1b
	Radius $\sqrt{10k^2-7}$	A1ft	2.2a
		(3)	
(b)	$x^2+(2x-1)^2+6kx-2k(2x-1)+7=0\Rightarrow...x^2+(pk+q)x+rk+s(=0)$	M1	1.1a
	$5x^2+(2k-4)x+(2k+8)\ (=0)$	A1	1.1b
	$(2k-4)^2-4\times5\times(2k+8)=0\Rightarrow k=...$	dM1	2.1
	Critical values = $7\pm\sqrt{85}$	A1	1.1b
	$k<"7-\sqrt{85}"$ or $k>"7+\sqrt{85}"$ o.e.	ddM1	3.1a
	$k<7-\sqrt{85}$ or $k>7+\sqrt{85}$ o.e.	A1	2.5
		(6)	
		(9 marks)	

Notes

(a)(i)

B1: $(-3k,\ k)$ o.e. Accept without brackets. May be written as $x=-3k,\quad y=k$

(a)(ii)

M1: Attempts to find r^2 by completing the square and collects terms outside the brackets on the other side of the equation. $(x\pm3k)^2-...k^2+(y\pm k)^2-...k^2+7=0\Rightarrow(x\pm3k)^2+(y\pm k)^2=\pm ak^2\pm b$

Alternatively, they may try to use general formulae such as

$x^2+y^2+2fx+2gx+c=0\Rightarrow r^2=f^2+g^2-c$

May also be implied by an expression for *r*.

A1ft: $\sqrt{10k^2-7}$ Condone unsimplified equivalent expressions such as $\sqrt{9k^2+k^2-7}$ and do not allow if this is written with the equation of the circle as $(x+3k)^2+(y-k)^2=10k^2-7$. It must be extracted from this and explicitly written as $\sqrt{10k^2-7}$ o.e.

Do not penalise if their square root does not go fully over all three terms as long as the intention is clear.

Only follow through on a centre of the form $(\pm3k,\ \pm k)$ which will lead to a radius of $\sqrt{10k^2-7}$

Do not allow $\pm\sqrt{10k^2-7}$ and do not isw e.g. if they divide their radius by 2 (thinking they had found the diameter) then A0

(b)

M1: Substitutes $y=2x-1$ into the equation of the circle or their manipulated equation of the circle from (a) and attempts to collect terms proceeding to $.x^2+(pk+q)x+rk+s=0$ where *p*, *q*, *r* and *s* are all non zero.

Condone arithmetical slips and do not be too concerned by the mechanics of their rearrangement.

May be implied by $5x^2+(2k-4)x+2k+8\ (=0)$ or by their values for *a*, *b* and *c* in their discriminant. Do not be concerned with the use of <, > or =

A1: $5x^2+(2k-4)x+2k+8\ (=0)$ (which may be implied by their *a*, *b* and *c* in their discriminant) Do not be concerned with the use of <, > or =

Check carefully the signs of $2k-4$ since $4-2k$ will lead to the same answers and should score maximum M1A0dM1A0ddM1A0

dM1: Attempts to find $b^2 - 4ac$ for their 3TQ and attempts to find at least one critical value. Do not be too concerned by the mechanics of their rearrangement.
If they find the root(s) directly from a calculator you will need to check this. (condone decimals which may be rounded or truncated)
It is dependent on the first method mark. Do not be concerned with the use of <, > or =

A1: $7 \pm \sqrt{85}$

ddM1: Attempts to find the outside region for their critical values. It is dependent on the previous two method marks. (Must have **two values** to be able to score this mark)
States e.g. $k < "7-\sqrt{85}"$, $k > "7+\sqrt{85}"$ (condone $k \geqslant "7+\sqrt{85}"$, $k \leqslant "7-\sqrt{85}"$).
Condone for this mark $x \leftrightarrow k$ and e.g. $"7+\sqrt{85}" \leqslant k \leqslant "7-\sqrt{85}"$. Allow any equivalent expression including set notation which includes both outside regions. Do not penalise poor notation to indicate the outside regions. Condone e.g. "and" o.e for this mark.

A1: $k < 7-\sqrt{85}$ or $k > 7+\sqrt{85}$ or any equivalent expression including set notation which includes **both** outside regions.
e.g. $k < 7-\sqrt{85}$, $k > 7+\sqrt{85}$ $\left(-\infty, 7-\sqrt{85}\right) \cup \left(7+\sqrt{85}, \infty\right)$
$\left\{k : k \in \mathbb{R}, k < 7-\sqrt{85}\right\} \cup \left\{k : k \in \mathbb{R}, k > 7+\sqrt{85}\right\}$.
Allow " ," "or", " $\cup$ " or a space between the answers (or on different line) but do not accept "and", " $\cap$ "
If a variable is used it must be in terms of *k*
Do not allow e.g. $"k < 7-\sqrt{85}$ and $k > 7+\sqrt{85}"$ or $\left[-\infty, 7-\sqrt{85}\right] \cup \left[7+\sqrt{85}, \infty\right]$
isw provided there is no contradiction with the correct answer.

Alternative method:
Using the formula for the perpendicular distance of a point from a line via a Further Maths method
Send to review if you are unsure how to mark these

M1: Substitutes the values of $2x - y - 1 = 0$ and $(-3k, k)$ into $d = \left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right|$

Condone sign slips.

A1: $(d =) \left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right|$

dM1: Attempts to proceed from $\left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right| < \sqrt{"10k^2 - 7"}$ to form a 3TQ (typically $k^2 - 14k - 36 > 0$) and attempts to find the critical values as above via any valid method. Do not be concerned with the use of <, > or =

A1ddM1A1: As above